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Characteristics of lump waves solutions in a (3+1)-dimensional nonlinear evolution equation

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Abstract— In this paper, we consider a (3+1)-dimensional nonlinear evolution equation to determine its periodic lump waves. To do that, we cast the equation into its Hirota bilinear form firstly. We offer periodic lump wave through a test function in-terms of exponential and periodic cosine functions. Finally, the interactions of solitary waves and lump waves are presented with an entire analytic derivation. Some grapgs are incorporated to visualize the dynamics of the obtain wave solutions.

Keywords: A (3+1)-dimensional nonlinear evolution equation; Hirota's bilinear form; lump wave; interaction phenomena.

1 Introduction

Nonlinear evolution equations have a lot of significant applications in different sides of mathematical physics and engineering. Generally, all the basic equations of physics are nonlinear and such types of nonlinear evolution equations (NLEEs) are often very difficult to get exact solution [1-29]. So, the powerful and effective methods to seek exact solutions of NLEEs still have very attraction to diverse group of researcher. The Darboux transformation [1], the tanh-function method [2], the extended tanh-function method [3], the homogeneous balance method [4], the Jacobi elliptic function expansion method [5], the F-expansion method [6], Hirota bilinear method [13, 18] and so on which many powerful and systematic approaches to obtain the exact solutions of NLEEs. Among those methods, the Hirota's bilinear method is rather heuristic and possesses significant features that make it practical for the determination of multiple soliton solutions, and for multiple singular soliton solutions [13] for an extensive class of NLEEs in a direct method.

Recently, we have seen two types of phenomena such as soliton fission and soliton fusion respectively [13] in many nonlinear science and engineering field such as the gas dynamics, laser, plasma physics, electromagnetic, and passive random walker dynamics [14]-[16]. Also, rogue wave solutions have drawn a big attention of mathematicians and physicists globally for amusing class of lump-type solutions. Such types of phenomena are found in different fields in physics such as plasmas, the deep ocean, nonlinear optic and even finance [17]-[19]. On the basis of Hirota bilinear forms, it is natural and interesting to hunt for rogue type solutions of NLEEs [21]-[22].

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The nonlinear conformable time-fractional PHI-four equations have been solved through the generalized Kudryshov method with conformable fractional derivative [23]. The new different varieties of soliton structures of the unstable nonlinear Schrodinger equations through the generalized Kudryshov method [24].

In this paper, we consider a (3+1)-dimensional nonlinear evolution equation to determine periodic lump waves. That's why we cast the equation into Hirota bilinear form firstly. Then we offer periodic lump wave through a test function in-terms of exponential and periodic cosine functions. Finally, the interactions of solitary waves and lump waves are presented with an entire analytic derivation. Some graphs are incorporated to visualize the dynamics of the obtain wave solutions.

2 Lump and solitary wave solutions to the Breaking Soliton equation

2.1 The bilinear form of (3+1)D nonlinear evolution equation

Consider the (3+1)-dimensional nonlinear evolution equation as

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{zz} = 0. \tag{1}$$

Through the dependent variable transformation as

$$u = 2(\ln f)_x, \tag{2}$$

Eq.(1) can be reduce to bilinear D operator form.

Substitute the equation (2) with $f = f(x, y, z, t)$ into equation (1) we obtain

$$(D_t D_y - D_x^3 D_y - 3D_x^2 + 3D_z^2) f.f = 0 \tag{3}$$

Where, $D_t D_y$, $D_x^3 D_y$, D_x^2 and D_z^2 are all the bilinear derivative operators [20] defined by

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$$D_x^{\alpha} D_y^{\beta} D_t^{\gamma} (\rho.Q) = (\frac{\partial}{\partial x} - \frac{\partial}{\partial x'})^{\alpha} (\frac{\partial}{\partial y} - \frac{\partial}{\partial y'})^{\beta}$$

$$\times (\frac{\partial}{\partial t} - \frac{\partial}{\partial t'})^{\gamma} \rho(x,y,t) Q(x' y' t')\Big|_{x'=x, y'=y, t'=t} \quad (4)$$

Using formula Eq.(4), Eq.(3)reduces to

$$ff_{xxxy} - f_{xxx} f_y - 3 f_x f_{xxy} + 3 f_{xx} f_{xy} = 0. \quad (5)$$

2.2 Lump wave solutions

Let us adopt that Eq. (5) has a ansatz in the following form:

$$f = l_1 \cos(a_1 x + a_2 y + a_3 z + a_4 t) + \exp(a_5 x + a_6 y + a_7 z + a_8 t) + l_2 \exp\{-(a_5 x + a_6 y + a_7 z + a_8 t) \quad (6)$$

where, a_i, $(1 \le i \le 8)$ are arbitrary constants to be determined later. Setting Eq. (6) into bilinear form Eq. (5), we obtain some polynomials which are functions of the variables x, y, z and t. Equating all the coefficient of $\cos$, $\sin$ and $\exp$ to be zero, we can obtain the set of algebraic equations for a_i, $(1 \le i \le 8)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters a_i:

Set-1:

$$a_1 = 0,\ a_4 = -\frac{3a_3^2}{a_2}, a_5 = \frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-3}\sqrt{P}, a_6 = 0,$$

$$a_7 = 4a_3{}^{-3}\sqrt{P} - \frac{\sqrt[3]{P}}{2a_2} \quad (7)$$

where, $P = (4a_2 a_8 + 4\sqrt{\frac{a_2^3 a_8^2 + 32a_3^3}{a_2}})a_2^2$ and

a_2, a_3, a_8 are arbitrary constants.

Therefore, substituting Eq. (7) and Eq. (6) along with Eq.(2) yields the following periodic lump wave solution,

$$u = 2(\ln f)_x, \quad (8)$$

where, $f = l_1 \cos(a_2 y + a_3 z - \frac{3a_3^2}{a_2} t) +$

$$\exp\{(\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-3}\sqrt{P})x - (\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-3}\sqrt{P})z + a_8 t\}$$

$$+ l_2 \exp\{-(\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-3}\sqrt{P})x + (\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-3}\sqrt{P})z - a_8 t\}$$

and $P = (4a_2 a_8 + 4\sqrt{\frac{a_2^3 a_8^2 + 32a_3^3}{a_2}})a_2^2$.

Fig.1 shows the sketch of lump waves occurs periodically for the values $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, (a) gives 3D views from which one can reveal the lump wave or one dimensional rogue wave feathers in the $xt-$plane at $y = z = 0$.

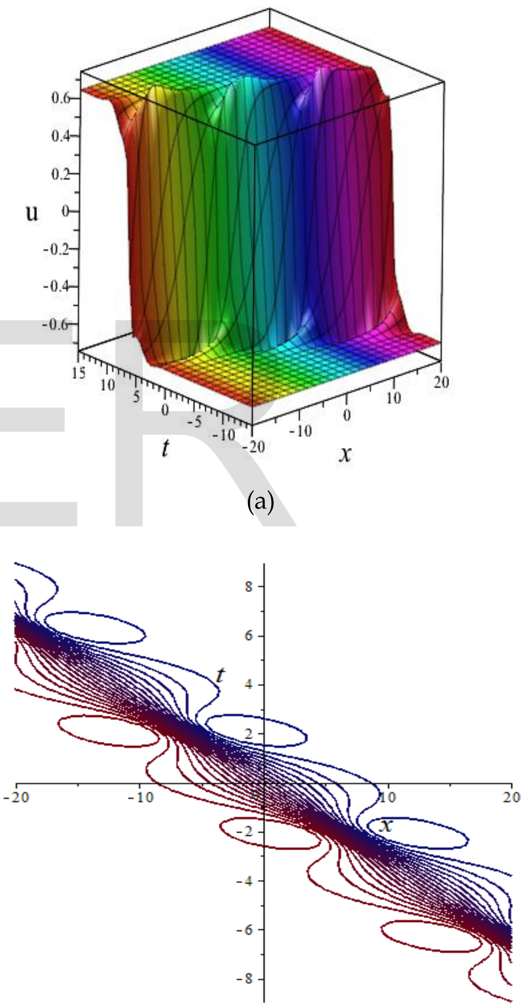


Figure-1: Lump wave solution (8) for Eq. (1) by choosing suitable parameters; (a) $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, Perspective view of the wave at $y = z = 0$. (b) $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, Corresponding contour plot of the wave.

It is also clear that the Fig.1 of Eq. (8) is the familiar eye-shaped lump wave solution which has a local deep whole and a height peak (clears from the views (b)) in each lump wave. Besides this, we discover that lump wave has the uppermost peak in its surrounding waves. The figures in the other plane exhibits similar characteristics but periodicity of lump may differ.

Set-2:

$$a_1 = 0,\ a_4 = -\frac{3a_3^2}{a_2}, a_5 = \frac{\sqrt[3]{P}}{2a_2} + 4a_3{}^{-\sqrt[3]{P}}, a_6 = 0,$$

$$a_7 = \frac{\sqrt[3]{P}}{2a_2} + 4a_3{}^{-\sqrt[3]{P}} \tag{7}$$

where, $P = (4a_2a_8 + 4\sqrt{\frac{a_2^3a_8^2 - 32a_3^3}{a_2}})a_2^2$ and

a_2, a_3, a_8 are arbitrary constants.

Therefore, substituting Eq. (9) and Eq. (6) along with Eq.(2) yields the following periodic lump wave solution,

$$u = 2(\ln f)_x, \tag{10}$$

Where, $f = l_1 \cos(a_2y + a_3z - \frac{3a_3^2}{a_2}t) +$

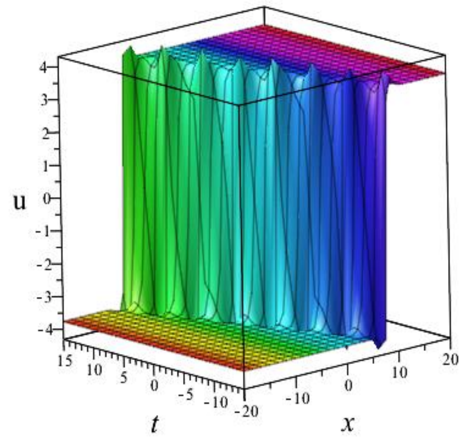
$\exp\{(\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-\sqrt[3]{P}})x + (\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-\sqrt[3]{P}})z + a_8t\} +$

$l_2 \exp\{-(\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-\sqrt[3]{P}})x - (\frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-\sqrt[3]{P}})z - a_8t\}$

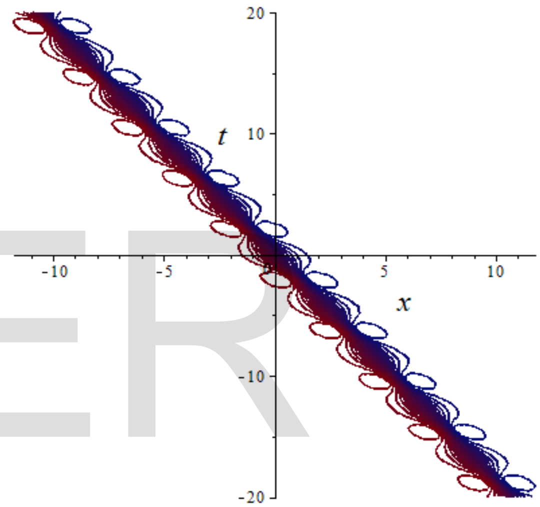
Where, $P = (4a_2a_8 + 4\sqrt{\frac{a_2^3a_8^2 - 32a_3^3}{a_2}})a_2^2$ and

a_2, a_3, a_8 are arbitrary constants.

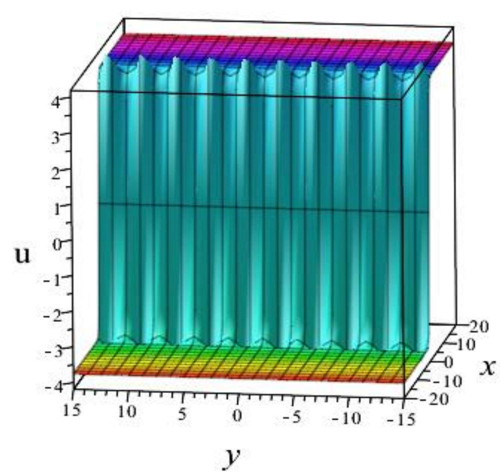
Fig.2 shows the sketch of lump waves occurs periodically for the values $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, (a) gives 3D views from which one can reveal the lump wave or one dimensional rogue wave feathers in the $xt-$plane at $y = z = 0$. It is also clear that the Fig.2 of Eq. (10) is the familiar eye-shaped lump wave solution which has a local deep whole and a height peak (clears from the views (b)) in each lump wave. Besides this, we discover that lump wave has the uppermost peak in its surrounding waves. The figures in the other plane exhibits similar characteristics but periodicity of lump may differ (see Fig-2(c) and (d) in the $xy-$plane).



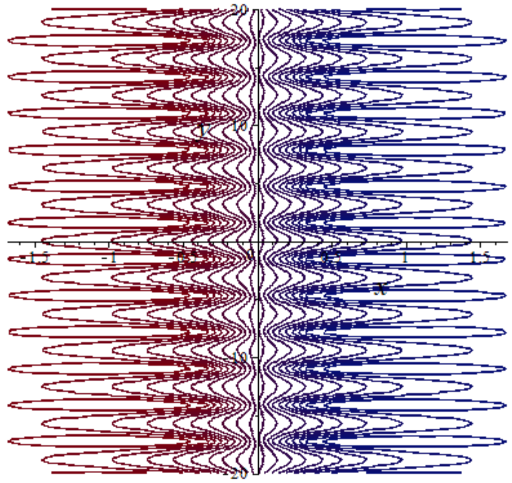
(a)



(b)



(c)



(d)

Figure-2: Lump wave solution (10) for Eq. (1) by choosing suitable parameters: $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$.(a) Perspective view of the wave at $y = z == 0$. (b) Corresponding contour plot of the wave.

4 Conclusion

In concluding remarks, based on the Hirota bilinear process, we have fruitfully offered two collision phenomena between a solitary type lump wave and a periodic cosine function solution to the (3+1)-dimensional nonlinear evolution equation equation. The lump wave comes in term of two exponentials and periodicity comes in term of cosine function and after collision the interaction exhibits as periodic breather type periodic lump waves. Also the results have been depicted graphically via 3D plot, contour plots to realize the real dynamics of the interactive waves. These outcomes will serve as a very significant milestone in the study of water waves in mathematical physics and engineering phenomena.

References

[1] C.H. Gu, H.S. Hu, Z.X. Zhou, Darboux Transformation in Soliton Theory and Its Geometric Applications, Shanghai Scientific Techinical Publishers, Shanghai, 1999.

[2] E. G. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A* 277, 212-218, 2000.

[3] Z. S. L, H. Q. Zhang, Applications of a further extended tanh method, *Appl. Math. Comput.* 159, 401-406, 2004.

[4] M. L. Wang, Y. B. Zhou, Z. B. Li, Applications of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Phys. Lett. A. 216, 67-75, 1996.

[5] S. K. Liu, Z. T. Fu, S. D. Liu, Q. ZHAO, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A. 289, 69-74, 2001.

[6] M. L. Wang, X. Z. Li, Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, Chaos, SolitonsFract. 24, 1257-1268, 2005.

[7] J. Manafian, M. Lakestani, A. Bekir, Study of the Analytical treatment of the (2+1)-Dimensional Zoomeron, the Duffing and the SRLW Equations via a new analytical approach, Int. J. Appl. Comput. Math. 2(2), 243–268, 2016.

[8] O. González-Gaxiola, R. Bernal-Jaquez, Applying Adomian Decomposition Method to Solve Burgess Equation with a Non-linear Source, Int. J. Appl. Comput. Math. 3 (1), 213–224, 2017.

[9] Zaid Odibat, A Riccati equation approach and travelling wave solutions for nonlinear evolution equations, Int. J. Appl. Comput. Math., 3 (1), 1-13, 2017.

[10] B. Bira, T. Raja Sekhar, Some exact solutions for sediment transport equations, Int. J. Appl. Comput. Math 1 (2), 251–255, 2015.

[11] M. Shakeel, S. T. Mohyud-Din, A novel (G'/G)-expansion method and its application to the (3 + 1)-Dimensional Burger's equations, Int. J. Appl. Comput. Math 2 (1), 13–24, 2016.

[12] M. B. Hossen, H. O. Roshid, M. Z. Ali, Modified double sub-equation method for finding complexiton solutions to the (1+1) Dimensional nonlinear evolution equations, Int. J. Appl. Comput. Math 3 (3), 1–19, 2017.

[13] H. O. Roshid, M.M. Rashidi, Multi-soliton fusion phenomenon of Burgers equation and fission, fusion phenomenon of Sharma–Tasso–Olver equation, Journal of Ocean Engineering and Science, 2 (2), 120-126, 2017.

[14] A. M. Wazwaz, (2+1)-Dimensional Burgers equations BE(m+n+1): Using the recursion operator. Appl. Math. Comput. 219, 9057-9068, 2013.

[15] H. O. Roshid, Novel solitary wave solution in shallow water and ion acoustic plasma waves in-terms of two nonlinear models via MSE method, Journal of Ocean Engineering and Science, 2 (3) , 196-202, 2017.

[16] H. O. Roshid, M. M. Roshid, N. Rahman, M. R. Pervin, New solitary wave in shallow water, plasma and ion acoustic plasma via the GZK-BBM equation and the RLW equation, Propulsion and Power Research, 6 (1) , 49-57, 2017.

[17] P. Muller, C. Garrett, A. Osborne, Meeting Report: Rogue Waves—The Fourteenth 'Aha Huliko'a Hawaiian Winter Workshop. Oceanography 18, 66-75, 2005.

[18] H.O. Roshid, Lump Solutions to a (3+1)-Dimensional Potential-Yu–Toda–Sasa–Fukuyama (YTSF) Like Equation, Int. J. Appl. Comput. Math (2017).

[19] C. Kharif, E. Pelinovsky, A. Slunyaey, Rogue Waves in the Ocean, Observation, Theories and Modeling, Springer, New York (2009).

[20] N. Akhmediev, A. Ankiewicz, M. Taki, Waves that appear from nowhere and disappear without trace, Phys. Lett. A 373, 675-678, 2009.

[21] B. L. Guo, L. M. Ling, Q. P. Liu, Nonlinear Schrödinger equation: Generalized Darboux transformation and rogue wave solutions. Phys. Rev. E 85 (2), 026607, 2012.

[22] C. Li, J. He, K. Porseizan, Rogue waves of the Hirota and the MaxwellBloch equations. Phys. Rev. E 87 (1), 012913, 2013.

[23] M. A. Rahman, M. N. Alam and H. O. Roshid, The Generalized Kudryshov Method Implemented to the Nonlinear Conformable Time-Fractional PHI-Four Equation, Annals of Pure and Applied Mathematics, Vol. 21, No. 1, 69-76, 2020. [DOI: http://dx.doi.org/10.22457/apam.v21n1a9661]

[24] M. A. Rahman and M. N. Alam "*The generalized Kudryshov method applied to the unstable nonlinear Schrodinger equation in mathematical physics*" International Journal of Mathematics Trends and Technology 66.3 (2020):82-89. [DOI: 10.14445/22315373/IJMTT-V66I3P512]